Can the lottery paradox be solved by identifying epistemic justification with epistemic permissibility?
Benjamin Kiesewetter, Lund University, 02.11.2015

1. THE LOTTERY PARADOX

The lottery paradox occurs when we combine two plausible claims about epistemic justification:

Probability claim: If A's evidence makes p exceedingly likely, then A has justification for believing p.
Conjunction claim: If A has justification for believing p, and A has justification for believing q, then A has justification for believing p&q.[1]

> *Clarification:* I use "A has justification for believing p" and "A can justifiably believe p" equivalently to refer to the fact that A has ordinary *outright* (propositional) justification for an ordinary *outright* belief in p.

The lottery paradox (Kyburg 1961, 197): A knows that he is confronted with a fair lottery with a large number of tickets n one and only one of which will win. Assuming both the probability and the conjunction claim, we are forced to accept the paradoxical conclusion that even though A knows that one ticket will win, A has justification for believing that all tickets will lose:

(1) $JBl_1 \& \ldots \& JBl_n$, from probability claim
(2) $(JBp \& JBq) \rightarrow JB(p \& q)$, conjunction claim
(3) $JB(l_1 \& \ldots \& l_n)$, from (1) and iterated applications of (2)

Since (3) is clearly false, either (1) or (2) must be rejected. Kyburg originally wanted to show that (2) is false. Some epistemologists have followed him, others have instead rejected (1).[2]

2. KROEDEL'S SOLUTION

Kroedel (2012) argues that one can avoid the paradoxical conclusion (3) while holding onto (1) and a *wide-scope* interpretation of (2):

The wide-scope conjunction claim: If A has justification for believing p and believing q, then A has justification for believing p&q. $[J(Bp \& Bq) \rightarrow JB(p \& q)]$

This solution requires, however, that the following principle is false:

Justification agglomeration: If A has justification for believing p, and A has justification for believing q, then A has justification for believing p and believing q. $[(JBp \& JBq) \rightarrow J(Bp \& Bq)]$

For, if justification agglomerates, Kroedel's assumptions entail the paradoxical conclusion again:

(1)$_{WS}$ $J(Bl_1 \& \ldots \& Bl_n)$, *from (1) by justification agglomeration*
(2)$_{WS}$ $J(Bp \& Bq) \rightarrow JB(p \& q)$, *wide-scope conjunction claim*
(3) $JB(l_1 \& \ldots \& l_n)$

Kroedel's argument is that since "justification is a species of permissibility", and "permissibility does not agglomerate", justification likewise fails to agglomerate.

Permissions don't agglomerate: From "It is permissible for A to marry Toni" and "It is permissible for A to marry Wlodek", it doesn't follow "it is permissible for A to marry Toni *and* marry Wlodek".

---

[1] The conjunction claim also follows from the more general *closure principle,* according to which we have justification for believing a proposition if it logically follows from other propositions each of which we can justifiably believe. I shall focus on the conjunction claim, however, since no stronger claim is needed to generate the paradox.

[2] See e.g. Foley (1979) and Klein (1985) for the rejection of the conjunction claim, and Ryan (1996) and Nelkin (2000) for the rejection of the probability claim.

3. First objection: The paradox re-emerges at the level of epistemic obligations

Plausibly, we are not obliged to believe everything we can justifiably believe.
- Harman (1986, 12–15): There is no point in cluttering one's mind with all the trivialities we can justifiably believe.
- Nelson (2010, 96–98): An obligation to believe everything we can justifiably believe is impossible to satisfy, because our evidence always justifies an infinite number of beliefs.

But justification could still give rise to obligation under circumstances that a lottery case might satisfy, along the lines of:

Bridge pattern: If A has justification for believing p, and condition C obtains, then A ought to believe p.

What's condition C?
- *Harman-inspired condition:* A is interested whether p is the case (cf. Harman 1986, 55).
- *Nozick-inspired condition:* A's expected utility of believing p is higher (or not lower) than the expected utility of having no belief about whether p is the case (cf. Nozick 1993, 86).
- *Attention condition:* A attends to p (Kiesewetter 2013, ch. 7.7).

If one of these or similar views are true, we might just stipulate a lottery case in which the relevant condition is satisfied and we get *obligations* to believe the relevant lottery propositions. We can then run the following argument to the paradoxical conclusion:

(4) $OBl_1 \& \ldots \& OBl_n$
(5) $O(Bl_1 \& \ldots \& Bl_n)$, from 4 by *obligation agglomeration* $[(OBp \& Obq) \rightarrow O(Bp \& Bq)]$
(6) $J(Bl_1 \& \ldots \& Bl_n)$, from 5 by *obligation implies justification* $[O(Bp) \rightarrow J(Bp)]$
(7) $JB(l_1 \& \ldots \& l_n)$, from 6 by *wide-scope conjunction claim* $[J(Bp \& Bq) \rightarrow JB(p \& q)]$

$\rightarrow$ Kroedel needs to reject any instance of the bridge pattern that, together with the probability claim, entails (4). But how?

4. Doxastic permissivism

Nelson ("We have no positive epistemic duties", 92): "My thesis is that there is nothing we positively ought to believe simply in virtue of our epistemic circumstances, and nothing that we 'ought' to believe at all, except given some further interest, desire, duty, or such like."

Three points:
1. In order to avoid infinitely many doxastic obligations, we need not assume that such obligations have *non-epistemic* background conditions. The attention condition is enough.
2. That doxastic obligations have non-epistemic *background conditions* does not entail that they are not epistemic obligations (compare moral background conditions for prudential requirements). The difference between epistemic and non-epistemic doxastic obligations should rather be seen as depending on whether non-epistemic *reasons* for belief play any role.
3. Even if we buy Nelson's full story, the lottery paradox re-emerges at the level of non-epistemic obligations to believe.

Kroedel seems committed to an even stronger claim:

Doxastic permissivism: No matter how carefully we attend to the question of whether p, and no matter how important that question is from a practical standpoint, as long as the probability of p is less than 1, we are always permitted to refrain from believing p.

Objection: This view undermines the practice of epistemic criticism. We criticise people not only for believing against their evidence but, at least sometimes, also for not-believing what the evidence makes very likely. Quite generally, criticism involves the assumption that an agent has failed to conform to a requirement and not merely omitted to make use of a permission.

Example: Consider Smith and Jones, two climate scientists who are both tied to political organisations that deny climate change. Both have very strong evidence for climate change, even though there is a residual probability that the data are misleading. Both are interested in the question of whether climate change is happening and pay sufficient attention to it. While Smith believes that climate change is not happening, Jones suspends judgement on that question and does not form a belief about climate change at all.
→ Smith and Jones are both rationally criticisable for their doxastic states. Jones does not escape such criticism merely by suspending judgment.

A general lesson:
- What generates the lottery (and also the preface) paradox is the fact that if we conjoin less-than-certain propositions in a conjunction, probability gets lost.
- Hence, in order to avoid the implication that we can justifiably believe unlikely conjunctions, we need to reject either (a) obligations to believe less-than-certain propositions, or (b) the assumption that one can justifiably believe the conjunction of all propositions that one is individually obliged to believe.
- The conjunction claim in both the wide-scope[3] and the narrow scope[4] reading entails (b), so Kroedel – *and everyone else who accepts a version of the conjunction claim* – has to reject (a) and adopt doxastic permissivism (which I have argued is quite implausible).

5. SECOND OBJECTION: JUSTIFICATION AGGLOMERATES

- *Thesis:* Even if doxastic permissivism were true, Kroedel's solution fails because epistemic justification agglomerates.
- *Logical point:* That practical permissions fail to agglomerate does not entail that epistemic permissions likewise fail to agglomerate.
- *Dialectical point:* Controversial cases like the lottery paradox aside, it is intuitive that epistemic permissions agglomerate. Kroedel has not given us any reason to question this intuitive principle.

Argument: Denying justification agglomeration commits one to the possibility that an agent is obliged to give up at least one of her individually justified beliefs. This obligation creates a dilemma.

Example: Lotta starts to form lottery beliefs. Starting from $B(l_1)$, she will at some point acquire a lottery belief $B(l_{max})$, such that if she adds a further lottery belief $B(l_{max+1})$ to her set of lottery beliefs,

---

[3] As is shown by the argument from (4) to (7).

[4] As is shown by the argument from (1) to (3), on the uncontroversial assumption that epistemic obligations entail epistemic justification.

she is no longer justified in having these beliefs together. By acquiring individually justified lottery beliefs, she finally arrives at the belief-set $\{B(l_1), \ldots, B(l_{max+1})\}$ that, according to Kroedel, she is not justified in having. Since Lotta is not justified, and thus not permitted, to have this set of beliefs, she is obliged *not* to have it. But how is she going to satisfy this obligation?

Widely accepted assumption: One can rationally form or revise beliefs only on the basis of sufficient object-given reasons, i.e. evidential considerations.

Dilemma, horn 1: Lotta has sufficient evidence for revising a lottery belief.
(i)   It is intuitively implausible that evidence that makes a proposition extremely likely is sufficient for revising a belief in that proposition.
(ii)   The probability claim (accepted by Kroedel) entails that Lotta's evidence is sufficient to *form* a lottery belief. Maintaining that the very same evidence is also sufficient to revise that belief commits one to rejecting the following plausible principle:

   • *Asymmetry principle:* If some body of evidence provides a sufficient basis for rationally forming the belief that p, then the same body of evidence does not provide a sufficient basis for rationally revising the belief that p.

Dilemma, horn 2: Lotta is subject to an epistemic obligation that she cannot rationally satisfy.
•   Accordingly, Lotta can satisfy her obligation only by irrational processes (such as wishful thinking) or non-rational processes (such as forgetting).
•   But even though epistemic obligations do not imply voluntary control over doxastic states, they surely imply some sort of 'can' that goes beyond the possibility of irrational or accidental conformity.

Generalization:
•   By denying justification agglomeration, one embraces the possibility of an epistemic obligation to give up an individually justified belief.
•   This creates a dilemma: either we have to assume that it can be fully rational to revise a belief on the same evidence on which it was rational to form that very belief, or we have to accept that there are epistemic obligations that cannot rationally be satisfied.
•   Better, then, to stick with *justification agglomeration!*

REFERENCES

Foley, R. 1979. "Justified Inconsistent Beliefs." *American Philosophical Quarterly* 16 (4): 247–57.
Harman, G. 1986. *Change in View. Principles of Reasoning.* Cambridge, MA: MIT Press.
Kiesewetter, B. 2013. *The Normativity of Rationality.* Ph.D. thesis, Humboldt University of Berlin. Forthcoming at: Oxford University Press.
Klein, P. 1985. "The Virtues of Inconsistency." *The Monist* 68 (1): 105–35.
Kroedel, T. 2012. "The Lottery Paradox, Epistemic Justification and Permissibility." *Analysis* 72 (1): 57–60.
Kyburg, H. E. 1961. *Probability and the Logic of Rational Belief.* Middletown: Wesleyan University Press.
Nelkin, D. K. 2000. "The Lottery Paradox, Knowledge, and Rationality." *The Philosophical Review* 109 (3): 373–409.
Nelson, M. T. 2010. "We Have No Positive Epistemic Duties." *Mind* 119 (473): 83–102.
Nozick, R. 1993. *The Nature of Rationality.* Princeton: Princeton University Press.
Ryan, S. 1996. "The Epistemic Virtues of Consistency." *Synthese* 109 (2): 121–41.